**Inequalities : The Tool Kit**

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Here are the basic inequalities which are very useful to solve any inequality. The inequalities are stated with some special cases.

1. **Triangle Inequality:** For all,,

**Equality:** Iff all have the same sign.

1. **inequality:** For all, ,

**Equality:** Iff all are equal.

**Weighted Inequality**:

If and , then,

**Equality:** Iff all are equal.

1. **Rearrangement Inequality:**

If we consider two sequence of real numbers,

For any permutation of we have that,

**Maximum and Minimum of Rearrangement inequality:**

So,

**Equality:** Iff (But the maximum minimum inequality always holds)

* **Chebyshev’s Inequality:**

Another form,

**Equality:** Iff there exists some with

1. **Cauchy-Schwarz Inequality:**

**Equality:** Iff there exists some with

1. **Helpful Inequality (Angel’s form):**  
   If and , then,

**Equality:** Iff

1. **Schur’s Inequality:**

**Equality:** Iff or two of are equal and other is

1. **Power Mean Inequality:**

If ; , and non-zero reals with , then,

Remark: With, here are nothing but the calssical inequalities,

1. **Weighted Power Mean Inequality:**

If are non-negative reals and , then,

is in general, a non-decreasing function of .

Remark: It can also produce the classical inequalities,

1. **Holder’s Inequality:**

If and such that, , then

Remark: With we get the famous Cauchy-Schwarz Inequality.

* More generally, if are positive real numbers such that . Then,
* Special case,

1. **Minkowski’s Inequality:**

If and then,

1. **Nesbit’s Inequality:** For ,

**Equality:** Iff

1. **Bernouli’s Inequality:** For all and ,
2. **Jesen’s Inequality:**

If is convex in , then for any with and , we have,

**Convexity Test:** Let be twice differentiable function on . Then,

* is convex on if for every .
* is **strictly convex** on if for every in the interior of .

1. **Some Important trivial Inequalities:**

**Equality:** Iff all variables are equal.

1. **Some very useful factorization techniques:**

*Equivalently,*